

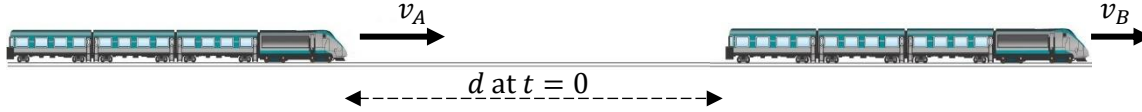


PHYS 101 – General Physics I Midterm Exam 1

Duration: 90 minutes

Saturday, 01 March 2025; 13:00

1. Two trains, labelled A and B, are traveling on the same track in the same direction, with train B ahead of train A. The speed of train A is v_A , while the speed of train B is v_B , with v_A being greater than v_B ($v_A > v_B$). At time $t = 0$, when the front end of train A is a distance d from the rear end of train B, train A begins to **decelerate** with constant acceleration of magnitude a ($a > 0$) to prevent a collision.



- (a) (7 Pts.) What is the distance between the two trains as a function of time for $t > 0$?
- (b) (8 Pts.) What is the relative velocity of train A with respect to train B as a function of time for $t > 0$?
- (c) (15 Pts.) What is the expression in terms of v_A , v_B and d for the minimum acceleration a , which avoids a collision?

Solution: (a) Positions of the two trains for $t > 0$ are

$$x_A(t) = v_A t - \frac{1}{2} a t^2, \quad x_B = d + v_B t.$$

Therefore, the distance between the two trains as a function of time for $t > 0$ is

$$x_B(t) - x_A(t) = d + (v_B - v_A)t + \frac{1}{2} a t^2.$$

(b) The relative velocity of train A with respect to train B as a function of time for $t > 0$ is

$$v_{AB} = v_A(t) - v_B(t) = v_A - at - v_B \rightarrow v_{AB} = (v_A - v_B) - at$$

(c) A collision is avoided if the relative velocity of the two trains becomes zero when the distance between the two trains is greater than or equal to zero. Therefore, the minimum acceleration a which avoids a collision is found as

$$v_{AB}(t_c) = 0 \rightarrow t_c = \frac{v_A - v_B}{a},$$

$$x_B(t_c) - x_A(t_c) = d + (v_B - v_A) \left(\frac{v_A - v_B}{a} \right) + \frac{1}{2} a \left(\frac{v_A - v_B}{a} \right)^2 = d - \frac{(v_A - v_B)^2}{2a},$$

so,

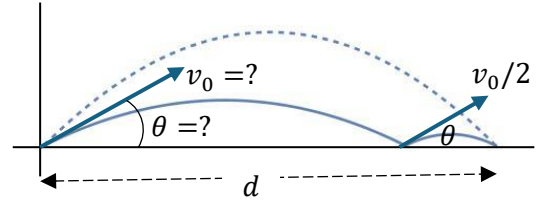
$$x_B(t_c) - x_A(t_c) \geq 0 \rightarrow a_{\min} = \frac{(v_A - v_B)^2}{2d}.$$

2. Given that the acceleration due to gravity is g , and the maximum range on a level ground of a tennis ball launcher (a machine which throws tennis balls) is d .

(a) (5 Pts.) What is the initial speed v_0 of the launched ball?

(b) (15 Pts.) Assume that a ball projected at an angle θ from the launcher with initial speed v_0 bounces off the ground with a speed $v_0/2$ at the same angle θ with the horizontal. What should the angle θ be to make the ball reach the total distance d with one bounce?

(c) (15 Pts.) Assuming that the ball is launched at time $t = 0$, at what time does it hit the ground for the second time?



Solution: (a) Position of the projectile at any time after it is fired is

$$x(t) = v_0 \cos \theta \, t, \quad y(t) = v_0 \sin \theta \, t - \frac{1}{2} g t^2.$$

On a level ground it hits the ground when

$$y(t_T) = 0 \quad \rightarrow \quad t_T = \frac{2v_0 \sin \theta}{g} \quad \rightarrow \quad x(t_T) = \frac{2v_0^2 \sin \theta \cos \theta}{g} = \frac{v_0^2}{g} \sin 2\theta.$$

The maximum range is reached when $\theta = \pi/4$, $\sin 2\theta = \sin \pi/2 = 1$, which means $d = v_0^2/g \quad \rightarrow \quad v_0 = \sqrt{gd}$.

(b) If the ball bounces off the ground with a speed $v_0/2$ at the same angle θ with the horizontal and the distance covered with one bounce is d , we have

$$d = \frac{v_0^2}{g} \sin 2\theta + \frac{(v_0/2)^2}{g} \sin 2\theta \quad \rightarrow \quad d = d \sin 2\theta + \frac{d}{4} \sin 2\theta \quad \rightarrow \quad \sin 2\theta = \frac{4}{5}, \quad \theta = \frac{1}{2} \arcsin\left(\frac{4}{5}\right).$$

(c) Since

$$\sin 2\theta = \frac{4}{5} \quad \rightarrow \quad \cos 2\theta = \frac{3}{5} \quad \rightarrow \quad 1 - 2 \sin^2 \theta = \frac{3}{5} \quad \rightarrow \quad \sin \theta = \frac{1}{\sqrt{5}}.$$

So

$$t_1 = \frac{2v_0 \sin \theta}{g} = \frac{2}{\sqrt{5}} \sqrt{\frac{d}{g}}, \quad t_2 = \frac{2(v_0/2) \sin \theta}{g} = \frac{1}{\sqrt{5}} \sqrt{\frac{d}{g}}.$$

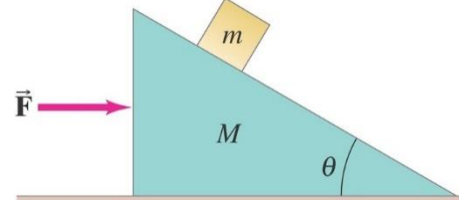
Hence, the ball hits the ground for the second time at time

$$t_T = t_1 + t_2 = \frac{3}{\sqrt{5}} \sqrt{\frac{d}{g}}.$$

3. A small block of mass m rests on the rough, sloping side of a triangular block of mass M and angle of inclination θ , which itself rests on a horizontal frictionless table as shown. The gravitational acceleration is g , and the coefficient of static friction between the small block and the triangular block is μ_s .

(a) (5 Pts.) Draw a free body diagram for the small block.

(b) (30 Pts.) Determine the minimum horizontal force F applied to M that will prevent the small block m to start sliding down the incline.



Solution:

(b) If the small block does not slide on top of the triangular block, both blocks accelerate with the same acceleration in the horizontal direction. Newton's second law is

$$n \sin \theta - f_s \cos \theta = ma, \quad n \cos \theta + f_s \sin \theta = mg.$$

Solving these two equations for the unknowns n and f_s , we get

$$n = ma \sin \theta + mg \cos \theta, \quad f_s = -ma \cos \theta + mg \sin \theta.$$

Since $|f_s| \leq \mu_s |n|$, we have

$$g \sin \theta - a \cos \theta \leq \mu_s (g \cos \theta + a \sin \theta) \rightarrow a \geq \left(\frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta} \right) g.$$

Since both blocks accelerate with the same acceleration, we have

$$F = (M + m)a \rightarrow a = \frac{F}{M + m}.$$

But depending on values of μ_s and θ the small block may not slide down on the triangular block even if the acceleration is zero. With $a = 0$, we have

$$n \sin \theta - f_s \cos \theta = 0, \quad n \cos \theta + f_s \sin \theta = mg \rightarrow n = mg \cos \theta, \quad f_s = mg \sin \theta,$$

and $|f_s| \leq \mu_s |n|$ means

$$mg \sin \theta \leq \mu_s mg \cos \theta \rightarrow \mu_s \geq g \tan \theta.$$

Hence the result is

$$F_{\min} = 0, \quad \text{if } \mu_s \geq g \tan \theta, \quad F_{\min} = \left(\frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta} \right) (M + m)g, \quad \text{if } \mu_s < g \tan \theta.$$

